

# TESTS OF GENERAL RELATIVITY AT LARGE DISTANCES AND DARK MATTER<sup>★</sup>

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## ABSTRACT

The most simple observed cases of gravitational lensing of distant quasars and galaxies by galaxies and clusters of galaxies are used to test Einstein's theory of General Relativity and Newtonian Gravity over galactic and intergalactic distances. They extend the distance range over which Newton-Einstein Gravity has been tested by 10 orders of magnitude. Although the precision of the tests are far from the precision of the solar system tests of EGR and those from pulsar timing in close binary systems, they confirm quite accurately the validity of Einstein's General Relativity and its weak field limit, Newtonian Gravity, over galactic and intergalactic distances and the existence of large quantities of dark matter in galaxies and clusters of galaxies. Future observations can improve the accuracy of the tests and reduce the possibilities for systematic errors.

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## Introduction

There are at least two good reasons for testing the validity of Einstein's General Relativity (EGR) and its weak field limit, Newtonian Gravity (NG), over cosmic distances;

(a) It has not been tested before over such distances - All astronomical tests of EGR and NG, so far, were limited to the solar system<sup>1</sup> and to close binary systems<sup>2-6</sup> (PSR 1913+16, 4U1820-30 and PSR 0655+64), i.e., to distance scales less than 50 Astronomical Units whereas EGR and NG have been applied to astronomical systems such as galaxies, clusters of galaxies, superclusters and the whole Universe, which are typically  $10^6 - 10^{15}$  times larger.

(b) The dark matter problem - All the dynamical evidence from galaxies, clusters of galaxies, superclusters and large scale structures that they contain vast quantities of non luminous dark matter<sup>7</sup> has been obtained using EGR or NG for such large astronomical systems. Thus far, in spite of extensive laboratory searches no conclusive evidence has been found for finite neutrino masses or for the existence of other dark matter particles beyond the standard model that can solve the dark matter problem. This has led some authors to question the validity of EGR and NG over large distances and to suggest<sup>9</sup> that perhaps EGR and NG are only approximate theories of gravity and that a correct theory of gravity will eliminate the dark matter problem. Indeed alternative theories<sup>10</sup> to General Relativity or modifications<sup>11-13</sup> of Newton's laws have been proposed in order to explain the observations without invoking dark matter.

The general success of EGR in explaining observations of gravitational lensing of quasars and galaxies by distant galaxies and clusters of galaxies<sup>14</sup> suggests that it is valid over cosmological distances. However, most of the theoretical studies of gravitational lensing were devoted to interpreting detailed gravitational lensing observations and to extracting astrophysical and cosmological information from these observations. In all these studies EGR has been assumed to be the correct theory of gravity but was not tested explicitly. In this note, however, I use only general features of gravitational lensing of distant quasars and galaxies by galaxies or cluster of galaxies, which are not sensitive to lens modelling, to test directly the validity of EGR and Newton's laws over galactic and intergalactic distances. The tests are similar to the historical 1919 and 1967 tests of EGR from the deflection of light by the Sun<sup>15,16</sup> and from the time delay suffered by radar signals in the gravitational field of the Sun<sup>17,18</sup>. Both in these historical solar system tests of EGR and in the galactic tests of EGR, one compares the measured and the predicted deflection and/or time delay of light by the gravitational field of a body whose mass has been determined from Kepler's third law applied to the motion of a test particle around it (the Earth around the Sun, a star, a globular cluster or a gas cloud around the center of a galaxy). We show that the simplest known cases of gravitational lensing of distant sources (quasars or faint blue galaxies) by galaxies and clusters of galaxies near the line of sight to these sources confirm the validity of EGR and its weak field limit, NG, over galactic and intergalactic distances and the existence of large quantities of dark matter in galaxies and clusters of galaxies. Alternative theories of gravity that claim to solve the dark matter problem must pass both the precision tests of EGR (the solar system tests and the tests from pulsar timing in close binary systems) and explain the observations of gravitational

lensing of quasars and galaxies by distant galaxies and clusters of galaxies .

## II. Gravitational Lensing

**A. Gravitational Light Deflection:** Einstein's theory of general relativity predicts that light which passes at an impact parameter  $b$  from a spherical symmetric mass distribution is deflected by an angle which, for small angles, is given approximately by

$$\alpha \approx \frac{4GM(b)}{c^2 b} , \quad (1)$$

where  $G$  is Newton's gravitational constant and  $M(b)$  is the mass interior to  $b$ . The mass  $M(r)$  enclosed within a radial distance  $r$  from the center is given by Kepler's third law  $M(r) \approx v_{cir}^2 r / G$ , where  $v_{cir}$  is the circular velocity of a mass orbiting at a distance  $r$  from the center. Consequently, spiral galaxies, which have flat rotation curves ( $v_{cir} \approx const.$ ) have  $M(r) \propto r$ ,  $\rho(r) \propto 1/r^2$ , and  $M(b)/b \approx \pi v_{cir}^2 / 2G$ , which give rise to a constant deflection angle independent of impact parameter,

$$\alpha = 2\pi \left( \frac{v_{cir}}{c} \right)^2 . \quad (2)$$

For large spiral galaxies,  $v_{cir} \sim 250 \text{ km s}^{-1}$  and  $\alpha \sim 1''$ . In elliptical galaxies, or clusters of galaxies, whose total mass distributions are well described by singular isothermal sphere distributions  $\rho(r) \approx (1/2\pi G)(\sigma_{\parallel}/c)^2$ , the squared circular velocity is replaced by  $v_{cir}^2 = 2\sigma_{\parallel}^2$ , where  $\sigma_{\parallel}$  is the one-dimensional line-of-sight velocity dispersion in the galaxy or the cluster, respectively. For a typical large elliptical galaxy with  $\sigma_{\parallel} \sim 200 \text{ km s}^{-1}$  the constant deflection angle is  $\alpha \sim 1.5''$  while for a rich cluster with  $\sigma_{\parallel} \sim 1000 \text{ km s}^{-1}$  the constant deflection angle is  $\alpha \sim 30''$ . Hence, the large optical telescopes, VLA and/or VLBI radio telescopes can be and have been used to discover and study gravitational lensing of quasars and galaxies by galaxies and clusters of galaxies.

EGR and Newton's laws can be tested over galactic and intergalactic distances by comparing the deflection of light which is extracted from these observations and the deflection of light which is predicted from the measured rotation curves or line-of-sight velocity dispersions in these systems, using the geometrical relation

$$\vec{\theta}_I = \vec{\theta}_s + \frac{D_{LS}}{D_{OS}} \vec{\alpha} , \quad (3)$$

where  $\vec{\theta}_s$  and  $\vec{\theta}_I$  are the angular positions of the source and the image, respectively, relative to the lens, and  $D_{OS}$  and  $D_{LS}$  are the angular diameter distances from the observer to the source and from the lens (the galaxy or the cluster of galaxies) to the source, respectively.

Thus, in order to extract the deflection angle from the angular positions of the source images one must know both the distance ratio  $D_{LS}/D_{OS}$  and the angular position of the source relative to the lens. In a Friedmann-Robertson-Walker universe the angular diameter distance between object A with redshift  $z_A$  and object B with redshift  $z_B$  is given by

$$D_{A,B} = \frac{2c}{H_0} \frac{(1 - \Omega_0 - G_A G_B)(G_A - G_B)}{\Omega_0^2 (1 + z_A)(1 + z_B)^2}, \quad (4)$$

where  $G_{A,B} \equiv (1 + \Omega_0 z_{A,B})^{1/2}$ ,  $H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$  is the Hubble constant and  $\Omega_0 \equiv \rho_0/\rho_c$  is the present density of the universe in critical density units,  $\rho_c \equiv 8\pi G/3H_0^2$ . Although the distance for large redshifts objects depends strongly on the cosmological model, the ratio  $D_{LS}/D_{OS}$  is independent of  $H_0$  and depends rather weakly on the cosmological model if  $z_L \ll z_s$ , and  $\Omega_0 + \lambda_0 \lesssim 1$ .

The angular position of the source must be deduced from the multiple image pattern (angular positions and relative magnifications) of the source which is produced by the lens. Generally, this requires a complicated inversion procedure and additional assumptions. However, for testing EGR we selected only the gravitational lensing cases where the lens is simple, the pattern-recognition is straightforward and the deflection angle can be read directly from the simple multiple image pattern:

**B. Einstein Rings, Crosses, and Arcs:** On the rare occasion that a lensing galaxy with a radially symmetric surface density happens to lie on the line-of-sight to a distant quasar it forms in the sky a ring image (Einstein Ring<sup>19,20</sup>) of the quasar around the center of the lensing galaxy, whose angular diameter is

$$\Delta\theta = 4\theta_r \approx 2 \frac{D_{LS}}{D_{OS}} \alpha \approx 4\pi \frac{D_{LS}}{D_{OS}} \left( \frac{\sigma_{\parallel}}{c} \right)^2. \quad (5)$$

Five Einstein rings, MG1131+0456, 1830-211, MG1634+1346, 0218+357 and MG1549+3047 were discovered thus far by high resolution radio observations<sup>14</sup> (see for instance Fig. 2), but, only for MG1634+1346 are the redshifts of both the lens and the ring image known, allowing a quantitative test of EGR. (When the source is slightly off center, the ring breaks into a pair of arcs, as actually observed for the ring image MG1634+1346 of a radio lobe of a distant quasar<sup>21,22</sup>.)

When the lens has an elliptical surface density and the line of sight to the source passes very near its center, the Einstein ring degrades into an ‘‘Einstein Cross’’, i.e., four images that are located symmetrically along the two principal axes<sup>14</sup> (and a faint fifth image at the center), with a mean angular separation between opposite images given approximately by Eq.5, as observed<sup>23–27</sup> in the case of Q2237+0305 (Fig. 3a).

When an extended distant source, such as a galaxy, lies on a cusp caustic behind a giant elliptical lens, such as a rich cluster of galaxies, it appears as an extended luminous arc on the opposite side of the lens<sup>28,29</sup>. The angular distance of the arc from the center of the lens is given approximately by the radius of the Einstein ring. Giant arcs were discovered, thus far, in the central regions of 13 rich clusters<sup>14</sup> (see for instance Fig. 4) and in six cases, Abell 370, 963 and 2390, Cl0500-24, Cl2244-02 and Cl0024+1654 the redshifts of both the giant arc image and the cluster are known and the velocity dispersion in the cluster has been estimated from the redshifts of the member galaxies or the X ray emission, allowing a quantitative test of EGR.

**C. Gravitational Time Delay:** In the thin lens approximation the time delay predicted by EGR is a sum of the time delay due to the difference in path length between deflected and undeflected light rays and the time delay due to the gravitational potential felt by the light rays<sup>14</sup>

$$\Delta t \approx (1 + z_L) \left[ \frac{D_{OL} D_{OS}}{2c D_{LS}} (\vec{\theta}_I - \vec{\theta}_s)^2 - \frac{\phi(\theta_I)}{c^3} \right], \quad (6)$$

where  $\phi(\theta_I)$  is the gravitational potential of the lens at  $\theta_I$ . Thus, the time delay between two images A,B, due to a lensing galaxy with nearly spherical isothermal mass distribution that lies near the line-of-sight to the source (even if it is embedded in a large cluster with an approximately constant deflection angle over the whole image), reduces to a simple form,

$$\Delta t_{A,B} \approx 2\pi(1 + z_L) (|\vec{\theta}_A| - |\vec{\theta}_B|) \left( \frac{\sigma_{\parallel}}{c} \right)^2 \frac{D_{OL}}{c}, \quad (7)$$

which can also be written as

$$\Delta t_{A,B} \approx (1 + z_L) (|\vec{\theta}_A| - |\vec{\theta}_B|) |\vec{\theta}_A - \vec{\theta}_B| \frac{D_{OS}}{D_{LS}} \frac{D_{OL}}{4c}. \quad (8)$$

Expression 8 is still valid when the lensing galaxy is embedded in a large cluster with an approximately constant deflection angle over the whole image. Note that while the deflection angle is dimensionless, i.e., depends only on dimensionless parameters, the time delay is dimensionfull and depends on the absolute value of the Hubble parameter (through  $D_{OL}$ ).

### III. Gravitational Lensing Tests of EGR

Figure 1 summarizes our comparison between the above EGR predictions and observations on the most simple known cases of gravitational lensing of quasars and galaxies by galaxies or cluster of galaxies. The error bars are statistical only and do not include model uncertainties. The comparison is described below in detail:

**A. The Einstein Ring MG1654+1346:** The ring image MG1654+1346 of a radio lobe of a distant quasar at redshift  $z_s = 1.75$ , formed by a bright elliptical galaxy at redshift  $z_L = 0.254$ , has<sup>21,22</sup> an angular diameter  $\Delta\theta = 1.97'' \pm 0.04''$ . The redshifts of the ring and lensing galaxy yield a distance ratio  $D_{LS}/D_{OS} \approx 0.73 \pm 0.01$ . Unfortunately, no published measurements are available of the one-dimensional velocity dispersion in the lensing galaxy. However, Langston et al.<sup>21,22</sup> measured a total B-band luminosity of the galaxy,  $L_B = (1.87 \pm 0.18) \times 10^{10} h^{-2} L_\odot$ , where  $h$  is the Hubble parameter in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $L_\odot$  is the luminosity of the Sun. The luminosities of bright elliptical galaxies were found by Faber-Jackson and Dressler to be correlated with the velocity dispersion within the inner few kiloparsecs of the galaxies<sup>30,31</sup> yielding for MG1654+1346 a velocity dispersion of  $\sigma_\parallel \approx 225 \pm 22 \text{ km s}^{-1}$ . Thus, Eq.5 predicts an angular diameter of  $\Delta\theta \approx 2.12'' \pm 0.42''$  for the Einstein ring, in good agreement with the observed diameter of  $\Delta\theta = 1.97 \pm 0.04''$ . A good description is also obtained for the detailed light pattern of MG1654+1346 assuming a constant mass to light ratio for the lens inside the Einstein ring.

**B. The Einstein Cross Q2237+0305:** This "Einstein Cross" is formed by a quasar with a redshift  $z_s = 1.695$  that lies behind the Zwicky galaxy that has a redshift  $z_L = 0.0394$  and a line-of-sight velocity dispersion of<sup>32</sup>  $209 \pm 19 \text{ km s}^{-1}$ . The redshifts of the lensed quasar and the lensing galaxy yield a distance ratio  $D_{LS}/D_{OS} \approx 0.95 \pm 0.01$ . The diameter of the ring is predicted by Eq.5 to be  $\Delta\theta \approx 2.3'' \pm 0.5''$ . It agrees with the high resolution observations of Yee<sup>24,25</sup> which gave  $\Delta\theta = 1.82'' \pm 0.03''$ , and with recent observations with the Hubble Space Telescope (Fig. 3a) with the JPL wide field camera which gave<sup>28</sup>  $\Delta\theta = 1.85'' \pm 0.05''$ . More accurate calculations of the image positions<sup>28</sup> which assume a mass distribution within the lensing galaxy proportional to its light distribution, yield their positions within 2% (see Fig. 3b), if the mass to blue light ratio is  $M/L_B = 12.3h$ , in good agreement with the usual observed mass to blue light ratio,  $M/L_B = (12 \pm 2)h$ , in E and SO galaxies<sup>33</sup>.

**C. The Einstein Arc In A370:** The giant arc subtends an angle larger than  $60^\circ$  at a radius of  $\theta_r = 26'' \pm 2''$  from the center of this rich cluster. The lensing cluster A370 and the giant arc have redshifts<sup>34,35-37</sup>  $z_L = 0.374$  and  $z_s = 0.724$ , respectively, yielding a distance ratio  $D_{LS}/D_{OS} = 0.392 \pm 0.008$ . Soucail et al. obtained a line-of-sight velocity dispersion for A370 of<sup>35</sup>  $\sigma_\parallel = 1700 \pm 170 \text{ km s}^{-1}$ . Fort et al. found<sup>37</sup>  $\sigma_\parallel = 1340_{-150}^{+230} \text{ km s}^{-1}$ , while Henry and Lavery found<sup>38</sup>  $\sigma_\parallel = 1587_{-214}^{+360} \text{ km s}^{-1}$ . The weighted mean value of these measurements is  $\sigma_\parallel = 1542_{-90}^{+140} \text{ km s}^{-1}$ , which is also consistent with the value  $\sigma_\parallel \approx 1500 \text{ km s}^{-1}$  that was estimated<sup>36</sup> from the total X-ray luminosity<sup>39</sup> of A370. Using the mean value, Eq.5 predicts an angular radius of  $\theta_r \approx 26.2'' \pm 7.1''$ , in good agreement with the observed radius,  $\theta_r \approx 26'' \pm 2''$ .

**D. The Einstein Arc In Cl2244-02:** The giant circular arc near the center of the rich cluster Cl2244-02, discovered by Lynds and Petrosian<sup>40</sup> subtends an angle of more than  $110^\circ$  at an angular distance of  $\theta_r = 9.9 \pm 0.2''$  from the

center of the cluster. The cluster and giant arc have redshifts <sup>41,42</sup>  $z_L = 0.328$  and  $z_s = 2.238$ , respectively, yielding a distance ratio  $D_{LS}/D_{OS} = 0.70 \pm 0.02$ . No published data are available yet on the velocity dispersion in Cl2244-02. However, it can be estimated from its measured X-ray luminosity, using a phenomenological relation between the X-ray luminosity and the velocity dispersion in rich clusters<sup>43</sup>. The X-Ray luminosity of Cl2244-02 was found to be<sup>44</sup>  $L_x \approx 3 \times 10^{44} h^{-2} \text{ erg s}^{-1}$  in the [0.5 - 4.5] keV band, yielding a velocity dispersion of  $\sigma_{\parallel} \approx 775 \pm 80 \text{ km s}^{-1}$ . With this value Eq. 5 predict an angular radius of  $\theta_r \approx 12'' \pm 3.1''$ , in good agreement with the observed radius,  $\theta_r \approx 9.9'' \pm 0.3''$ .

**E. The Einstein Arc In Cl0024+1654:** The giant arc discovered by Koo<sup>45</sup> near the center of the cluster Cl0024+1654, subtends an angle of more than  $40^\circ$  at an angular distance of  $\theta_r \approx 35''$  from the center of the cluster. The cluster has a redshift of  $z_L = 0.39$ . The redshift of the the arc  $z_s$  is not known, but it is estimated to be between 1 and 2, based on the absence of any emission lines in the spectrum<sup>46</sup> yielding a distance ratio  $D_{LS}/D_{OS} = 0.48 - 0.64$ . The line-of-sight velocity dispersion in Cl10024+1654 was estimated<sup>47,48</sup> to be  $1290 \pm 100 \text{ km s}^{-1}$ , yielding a radius of  $\theta_r \approx 20'' - 36''$ , in agreement with the observed radius  $\theta_r \approx 35''$  provided that the image has a redshift close to 2. (The segmentation of the arc is probably due to two galaxies near its central segment which perturb locally the smooth singular isothermal sphere potential but do not change significantly the average radial distance of the arc from the center of the cluster).

**F. Time Delay In The Double Quasar Q0957+561:** Thus far an observed time delay between different images of the same quasar has been established only for the double quasar Q0957+561. Although the lens system is not completely understood<sup>14</sup>, an approximate test of EGR is still possible. The lens is essentially a giant elliptical galaxy embedded in a large cluster<sup>49,50</sup> and produces two quasar images, A,B, at angular distances<sup>51</sup>  $|\vec{\theta}_A| \approx 5.218''$  and  $|\vec{\theta}_B| \approx 1.045''$ , respectively, from the center of the lensing galaxy, with an angular separation  $|\vec{\theta}_A - \vec{\theta}_B| \approx 6.175''$ . The lensing galaxy has a redshift  $z_L = 0.355$  and the images have a redshift  $z_s = 1.41$  which yield a distance ratio  $D_{OL}D_{OS}/D_{LS} \approx (0.365 \pm 0.015)c/H_0$ . From the predicted (Eq.8) and the observed<sup>51-56</sup> time delay between the two images,  $\Delta t = 1.48 \pm 0.03 Y$  and<sup>57,58</sup>  $\Delta t = 1.1 - 1.3 Y$ , if it is due to the giant galaxy, one obtains

$$H_0 \approx (76 \pm 4)(1Y/\Delta t_{A,B}) \approx 50 - 70 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (9)$$

This value is consistent with the known value of  $H_0$  from a variety of other measurements<sup>59</sup> ( $\approx 50 - 80 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ). Detailed modeling of the lens yields similar results<sup>14</sup>.

#### IV. Mass To Light Ratios

The mass interior to an Einstein ring, i.e., interior to  $b = \theta_r D_{OL}$  that follows from Eqs. 1-3 is given by

$$M(b) \approx \frac{c^2 \theta_r^2 D_{OL}}{4G} \frac{D_{OS}}{D_{LS}}. \quad (10)$$

From Eq.10 it follows that  $M(b) \approx (1.12 \pm 0.04) \times 10^{10} h^{-1} M_\odot$  interior to  $b \approx 0.5 h^{-1} kpc$  for 2237+0305, and  $M(b) \approx (9.0 \pm 0.4) \times 10^{10} h^{-1} M_\odot$  interior to  $b \approx 2.5 h^{-1} kpc$  for MG1654+1346. These masses and the measured luminosities inside the rings give  $M/L_B \approx (16 \pm 2)h$  for MG1654+1346 and  $M/L_B \approx (10 \pm 3)h$  for Q2237+0305 (in  $M_\odot/L_\odot$  units) in good agreement with the best dynamical  $M/L_B$  measurements for the cores of bright elliptical galaxies<sup>33</sup>,  $M/L_B = (11.8 \pm 2)h$ .

Although the observations of 2237+0305 and MG1654+1346 confirm the validity of EGR and Newtonian gravity for small galactic distances, and yield gravitating masses equal, within error bars, to the dynamical masses, i.e., to the masses derived from Newtonian dynamics, they shed no light on the dark matter problem because of the relatively small impact parameters that were probed. Rotation curves and X-ray studies of individual galaxies suggest that most of the dark matter in galaxies lies beyond such impact parameters<sup>61</sup>. In order to explore dark matter one must examine more massive and more distant lenses. Such cases are provided by 957+561, A370, Cl2244-02 and Cl0024+1654.

**Q937+561:** If the splitting between the two images of Q957+561 is due totally to the deflecting galaxy, then

$$M(b) \approx \frac{c^2 \theta_A |\vec{\theta}_A - \vec{\theta}_B| D_{OL}}{8G} \frac{D_{OS}}{D_{LS}} \approx (2.1 - 2.3) \times 10^{12} h^{-1} M_\odot \quad (11)$$

interior to impact parameter  $b \approx D_{OL} |\vec{\theta}_A| \approx (17 \pm 0.8) h^{-1} kpc$ . The V-magnitude of the lensing galaxy yields<sup>62</sup> a total luminosity of  $L \approx 7.6 \times 10^{10} h^{-2} L_\odot$ , and a mass-to-light ratio of  $M/L \approx 23h$  interior to  $b \approx 17 h^{-1} kpc$ .

The relative time delay and the angular positions of the two images yield a mass

$$M(b) \approx \frac{c^3 \Delta t}{2G(1+z_L)} \frac{\theta_A}{||\vec{\theta}_A| - |\vec{\theta}_B||} \approx 4.45 \times 10^{12} M_\odot \quad (12)$$

and a mass-to-light ratio of  $M/L \approx 58h^2$  interior to impact parameter  $b \approx 17 h^{-1} kpc$ . The analysis of velocity dispersions from stellar absorption measurements in elliptical galaxies leads to values of  $M/L \lesssim 14h$  independent of radius out to one effective radius<sup>61</sup>. HI rotation curves have been studied up to radii of about  $16 h^{-1} kpc$  for a few hydrogen rich ellipticals. They indicate<sup>61</sup> mass-to-light ratios somewhat larger,



but not significantly larger, approaching these radii. X-ray studies of giant ellipticals gave<sup>61</sup>  $M/L > 40h$  for radii between  $16h^{-1}kpc$  and  $50h^{-1}kpc$ . Thus, the lensing mass of 957+561 within  $17h^{-1}kpc$  and the mass-to-light ratio are consistent with those derived from HI rotation curves and X-ray studies of giant elliptical galaxies and they provide evidence for a large amount of dark matter in 957+561, in particular if  $H_0 \gg 50 km s^{-1}Mpc^{-1}$ .

**A370:** The gravitating mass enclosed within the impact parameter  $b = D_{OL}\theta_r \approx (85 \pm 5)h^{-1}kpc$  from the center of the cluster that follows from Eq.10 is  $M(b) \approx (1.54 \pm 0.10) \times 10^{14}h^{-1}M_{\odot}$ . This value yields a mass-to-light ratio of  $M/L_R \approx 130h$  within  $(85 \pm 5)h^{-1}kpc$ . Taking into account the difference between the mean galactic B - R index and the solar one, it gives  $M/L_B \approx 350h$ . Similar ratios were obtained from the virial theorem for the centers of other rich clusters indicating the existence of large quantities of gravitating dark matter in rich clusters of galaxies.

**Cl2244-02:** The gravitating mass enclosed within the impact parameter  $b = D_{OL}\theta_r \approx (31 \pm 2)h^{-1}kpc$  from the center of the cluster which follows from Eq. 10 is:  $M(b) \approx (1.15 \pm 0.10) \times 10^{13}h^{-1}M_{\odot}$ . It yields a mass-to-light ratio of about  $M/L_V \approx 200h$  in good agreement with the mass-to-light ratios found in other similar clusters from the virial theorem and from X-ray observations. Like in the case of A370, this mass-to-light ratio indicates that the cluster contains a large amount of gravitating dark matter.

## V. Conclusions

Observations of the six most simple known cases of gravitationally lensed images of quasars and galaxies, the Einstein Cross Q2237+05, the Einstein Ring MG1654+1346, the double quasar Q0957+561 and the Einstein Arcs in A370, Cl2244-02, and Cl0024+1654 confirm the predictions of Einstein's theory of General Relativity for the deflection and time delay in the gravitational field of galaxies and clusters of galaxies with masses equal to their dynamical masses (masses deduced from observed velocities using Newton's laws). These observations confirm within error bars the consistency between Einstein's General Relativity and Newtonian dynamics within impact parameters of  $\sim 0.5h^{-1}, 2.5h^{-1}, 17h^{-1}, 31h^{-1}, 85h^{-1}$  and  $117h^{-1} kpc$ , respectively. The lensing masses in galaxies and clusters are the same within error bars as their dynamical masses and are similar to the masses of other similar galaxies and clusters. The mass-to-light ratios in the lenses indicate the existence of large amounts of gravitating dark matter in the lenses within impact parameters larger than  $10h^{-1} kpc$ . In fact the gravitational lensing tests of EGR confirm that our Universe consists mainly of a mysterious gravitating dark matter. Detailed theoretical and observational studies of gravitational lenses can both improve the precision of the tests of EGR at large distances and shed some light on the distribution<sup>62</sup> and the nature of dark matter in galaxies, in clusters of galaxies and in the intergalactic space.

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## FIGURE CAPTIONS

Fig. 1: The ratios between the EGR prediction and the observations of the deflection and time delay of light from distant quasars and galaxies by galaxies or clusters of galaxies, displayed at the impact parameter of the deflected light relative to the center of the lens, for the Einstein Cross Q2237+05, the Einstein Ring MG1654+1346, the double quasar Q0957+561 and the Einstein Arcs in A370, Cl2244-02, and Cl0024+1654. The estimated errors in the ratios include the quoted observational errors and the errors in the theoretical estimates due only to errors in measured parameters and the absence of precise knowledge of  $\Omega$  and  $h$ , but not systematic errors.

Fig. 2: Radio images at 15 GHz and 5 GHz of MG1131+O456, the first discovered Einstein ring (by Hewitt et al 1988).

Fig. 3a: A 700 s exposure of the Einstein cross Q2237+0305 taken with the Wide Field Camera through the F336W filter on board the Hubble Space Telescope<sup>27</sup>.

Fig. 3b: A comparison between the observed (crossed boxes) and predicted (circles) positions of the four images of Q2237+0305 assuming a constant mass to light ratio near the center of the lensing galaxy<sup>27</sup>.

Fig. 4: The giant arc image of a distant blue galaxy formed by the rich cluster Abell 370 is the first discovered Einstein arc. It was discovered independently by Lynds and Petrosian (1986) and by Soucail, Fort, Mellier and Picat (1987).

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